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TRANSIENT FAULT CURRENTS IN AIRCRAFT D-C SYSTEMS

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ABSTRACT

Two simultaneous differential equations have been derived which satisfactorily describe the transient fault current in a shunt-wound aircraft d-c generator in terms of easily obtainable parameters. Improved accuracy of the result is attributed to eliminating the assumption of constant field flux linkages during the transient and substituting the assumption of constant rates of change of field flux linkages with armature and field currents. The field flux leakage coefficient was assumed constant. Theoretical and empirical transients compared favorably for both separately and self-excited generators. The effect of a carbon-pile voltage regulator was successfully predicted. Transient resistance has been redefined in such a manner that its value can be predicted in terms of the physical parameters of the generator.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

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TRANSIENT FAULT CURRENTS IN AIRCRAFT D-C SYSTEMS

INTRODUCTION

The steady-state characteristics of d-c generators have been discussed at length in various standard texts. More recently these characteristics have been expressed in a mathematical form which may be readily manipulated.[1] However, problems such as system protection and circuit-breaker design have made it desirable to understand more fully the behavior of d-c generator-regulator systems under transient conditions, e.g., the application or removal of a fault. Some work along these lines has already been attempted[2,3,4,5,6] with a certain degree of success, but in every case severe limitations have been imposed upon the conditions under which the analyses were valid. In discussing transient fault current Linville and Ward[7] assumed that the fault had zero resistance, that there was no regulator in the circuit, and that shunt field flux linkages remained constant for the range of time up to and beyond the peak surge current. The latter assumption presupposes that the resistance of the shunt field is negligibly small, or rather that the time constant of the field circuit is very large. The changes of effective air-gap flux during the transient have either been neglected or considered to change as a result of leakage changes.[8]

There is need for improvement of our understanding of the transient phenomena. An attempt has been made to relate the transient with the better-understood steady-state performance by choosing as many parameters as possible from among the measurable steady-state characteristics. It seemed logical at least to try out the assumption that field flux linkages vary with time. For purposes of simplicity and understanding it was considered worthwhile to assume parameters constant throughout the transient. Separate and self-excitation of the generator were both considered, and a method devised for predicting the effect of a carbon-pile voltage regulator. The resistance value of a fault affects the resulting transient and this is taken into account by introducing fault resistance in the boundary conditions and as a parameter of the circuit. Variations of transient load current with speed and initial load are automatically taken into account since the parameters used in this work are those measured at the condition existing prior to the application of the fault. To facilitate understanding of symbols used in this work a list of nomenclature has been appended.

[1] *Van Valkenburg, E. S., "Steady State Analysis of Aircraft D-C Generators," NRL Report No. E-3130, June 1947*

[2] *Linville, T. M., Ward, H. C., Jr., "Solid Short Circuit of Direct-Current Motors and Generators," AIEE Trans. 68, Pt. 1; 119-124, 1949*

[3] *McClinton, A. T., Brancato, E. L., Panoff, R., "Transient Characteristics of D-C Motors and Generators," AIEE Trans., 68, Pt. 2; 1100-1106, 1949*

[4] *Scorgie, D. G., "Transient Analysis of Voltage-Regulated Aircraft D-C Systems," NRL Report No. 3541, September 26, 1949*

[5] *Kaufmann, R. H., Finison, H. J., "D-C Power Systems for Aircraft," GE Review 48; 22-26 September 1945*

[6] *Yermolin, N. P., "Short Circuits of D-C Generators," Elektrichestvo 7; 26-32, July 1948*

[7] *Linville, T. M., loc. cit.*

[8] *Yermolin, N. P., loc. cit.*

BASIC EQUATION FOR THE ARMATURE CIRCUIT

The analysis gives in closed form the load current as a function of time when a fault resistance, R_L, is suddenly applied to the system. Results which have been obtained indicate that for this purpose two simultaneous linear differential equations can be written to describe the current surge satisfactorily for a simple shunt generator. Linearity is not as unreasonable an assumption as one might expect even though a d-c generator is highly nonlinear. This is evident if one considers that during the time the load current is rising the shunt-field current is also rising, and the sum of their mmf upon the field-iron circuit is decreasing by a relatively small amount. Thus, we have abandoned the notion of constant field flux linkages and replaced it by the assumption that at least while the load current is rising the change in mmf is small emough so that the field pole iron remains on a fairly linear portion of its saturation curve. It was demonstrated that the generated voltage decreases as much as 30 percent by the time peak surge current is reached and that this can be traced to a corresponding decrease in flux linking the shunt-field winding.

We shall assume that our fault is initiated by closing a switch across a portion of the existing load at time $t = 0$, leaving some remaining fault resistance, R_L, (Figure 1a). The armature circuit is then assumed to consist of a resistance $(R_a + R_L)$, an inductance, L_a, and two equivalent perfect generators all connected in series (Figure 1b). R_a and L_a are the resistance and incremental self-inductance, respectively, of the armature, plus any compensating and commutating windings. The first of the equivalent generators is a voltage step introduced at $t = 0$ and equal to the voltage across the switch before closing. Since the circuit is not passive a second equivalent generator must be inserted, the voltage of which is equal to the change of the machine's generated voltage as a function of time. In other words the latter equivalent generator voltage is given by:

$$e_g = \frac{\partial e_g}{\partial i_a} i_a + \frac{\partial e_g}{\partial i_f} i_f \tag{1}$$

where i_a is an incremental change from the initial value of load current,
i_f is an incremental change from the initial value of field current.

Throughout this report lower case i and e will be used to indicate increment from the initial currents and voltages, respectively. Replacing the partial derivatives by different symbols Equation (1) becomes:

$$e_g = D_a i_a + K_s i_f \tag{1'}$$

D_a is the partial derivative of the generated voltage with respect to the armature current at the initial operating condition, and represents the demagnetization due to armature reaction and cross ampere turns. It is considered constant and will usually be negative. K_s is the slope of a load saturation curve at the initial operating point and will be a positive constant.

Let E_o and $(I_a)_o$ be the terminal voltage of the machine and its load current, respectively, prior to the application of a fault. By definition we let

$$E_o' = E_o - (I_a)_o R_L,$$

so E_o' represents the step driving voltage introduced at t = 0. The basic equation of the armature circuit during the transient is therefore:

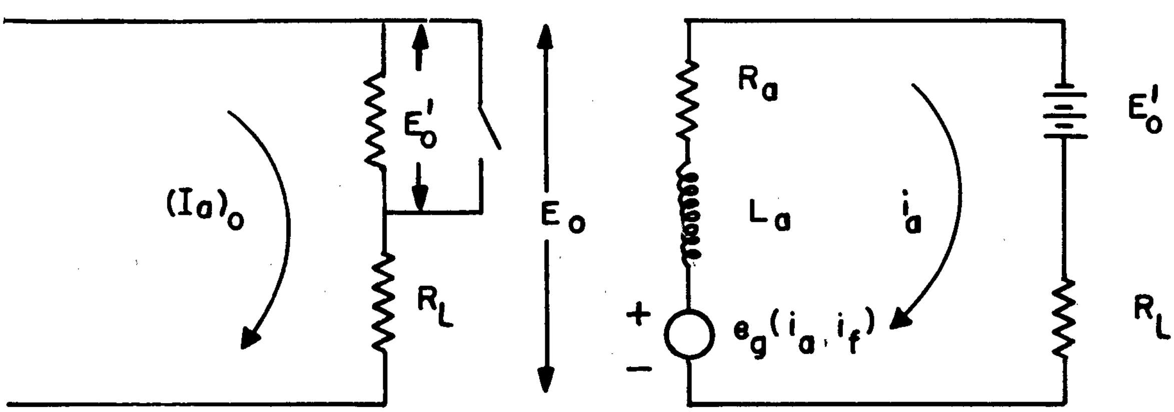
$$E_o' + e_g(t) = (R_a + R_L)\, i_a(t) + L_a \frac{di_a}{dt}, \qquad (2)$$

which, using Equation (1'), may be rewritten:

$$E_o' = (R_a + R_L - D_a)\, i_a(t) + L_a \frac{di_a}{dt} - K_s i_f(t) \quad . \qquad (2')$$

BASIC EQUATION FOR THE FIELD CIRCUIT

Equation (2'), which pertains to the armature circuit, holds whether or not a voltage regulator is included in the system, and for both separately and self-excited generators. Each of these four cases must be considered individually however when writing the field-circuit equation. In each case the symbol M is used to indicate an inductive coupling between armature circuit and the field winding. M is defined simply as $N_f \frac{\partial \phi_f}{\partial i_a}$ where ϕ_f is the flux linking the shunt field windings. M is not considered as a mutual coupling coefficient in the usual sense; and it will, ordinarily, be a negative number. Even though the comparable term may be left out of Equation (2'), this coupling must be included in the field equation. Four field-circuit equations were obtained.



(a) Prior to fault *(b) After fault*

Figure 1 - Equivalent circuit during transient

Case 1. No Voltage Regulator, Self-Excitation

Let the incremental self-inductance of the shunt field be L_f, and the field-circuit resistance be R_f. The change of terminal voltage after the fault is applied is given by $-E_o' + R_L i_a(t)$, and the transient field circuit equation is:

$$R_L i_a(t) - E_o' = L_f \frac{di_f}{dt} + M \frac{di_a}{dt} + R_f i_f(t) . \qquad (3)$$

Case 2. No Voltage Regulator, Separate Excitation

The transient field-circuit equation is given by:

$$0 = L_f \frac{di_f}{dt} + M \frac{di_a}{dt} + R_f i_f(t) . \qquad (4)$$

Case 3. Carbon-Pile Voltage Regulator Operating, Self-Excitation

Provided the fault resistance is small enough so that the generator terminal voltage drops more than a volt or two, the carbon-pile resistance can be assumed to change instantly to its minimum value. The field-circuit resistance changes instantly from its initial value, $(R_f)_o$, to its minimum value, R_s. This assumption appears valid both in theory[9] and in practice. Calculations from Mills show that if, for instance, the change of terminal voltage at $t = 0$ were 30 volts, then the rate of change of carbon-pile resistance would be approximately 15,000 ohms per second for a typical regulator. Oscillographic records have confirmed this, therefore the portion of the field circuit comprised of minimum resistance, R_s, and the field inductance experiences a change of voltage equal to $-E_o'' + R_L i_a(t)$. The difference between the voltages across this portion of the field the instant before and the instant after the fault is expressed by:

$$E_o'' = \frac{R_s}{(R_f)_o} E_o - (I_a)_o R_L$$

The field-circuit equation in this case is given by:

$$R_L i_a(t) - E_o'' = L_f \frac{di_f}{dt} + M \frac{di_a}{dt} + R_s i_f(t) . \qquad (5)$$

Case 4. Carbon-Pile Voltage Regulator Operating, Separate Excitation

Here again the same assumption can be made regarding the instantaneous change of carbon-pile resistance. The voltage change experienced by the remainder of the field is given by $-V = E\left(1 - R_s/(R_f)_o\right)$ where E is the constant excitation voltage. In this case the field circuit equation is given by:

[9] *Mills, R. L., "Dynamic Characteristics of Carbon Pile Voltage Regulators," NRL Report 3519 (Unclassified), Sept. 10, 1949*

$$-V = L_f \frac{di_f}{dt} + M \frac{di_a}{dt} + R_s i_f(t). \tag{6}$$

SOLUTION OF EQUATIONS FOR FAULT CURRENT

Equation (2') may easily be solved simultaneously with either Equation 3, 4, 5, or 6. The Laplace Transform of the total load current in all four cases will have the form:

$$I_a(s) = \frac{(I_a)_o}{s} + \frac{E_o'}{L_a s}\left[\frac{s + \beta_j}{(s+A)^2+B_j^{\,2}}\right] \tag{7}$$

where

$$A = 1/2\left[\alpha + \beta\right]$$

$$\alpha = \frac{R_a+R_L}{L_a}$$

$$\beta = \frac{R_f}{L_f}$$

B_j will be defined below.

The subscript, j, equals, 1, 2, 3, or 4 and refers to the four conditions under which the field circuit equation was written. The corresponding constants are defined as follows:

Case 1. No Regulator, Self-Excitation

$$\beta_1 = \beta - \frac{K_s}{L_f},$$

$$B_1 = \sqrt{\beta\,\alpha' - \frac{K_s R_L}{L_a L_f} - A^2}, \text{ where } \alpha' = \alpha - \frac{D_a}{L_a}.$$

Case 2. No Regulator, Separate Excitation

$$\beta_2 = \beta$$

$$B_2 = \sqrt{\beta\,\alpha' - A^2}.$$

Case 3. Voltage Regulator Operating, Self-Excitation

$$\beta_3 = \beta - \frac{K_s}{L_f}\,\frac{E_o''}{E_o'}$$

$$B_3 = B_1.$$

Case 4. Voltage Regulator Operating, Separate Excitation

$$\beta_4 = \beta = \beta_2$$

$$B_4 = B_2$$

It has been shown[10] that $M = \frac{D_a L_f}{K_s}$. Therefore in deriving Equation (7) from the differential equations the inductive coupling coefficient was eliminated by using the other more easily determined constants.

Transient load current as a function of time is given by the inverse transform of Equation (7), namely

$$i_a(t) = E_o' \left[K_1 + K_2\, e^{-At} \sin\,(B_j t + \psi)\right] \tag{8}$$

or

$$I_a(t) = (I_a)_o + E_o' \left[K_1 + K_2\, e^{-At} \sin\,(B_j t + \psi)\right] \tag{8'}$$

where

$$\psi = \tan^{-1}\left(\frac{B_j}{\beta_j - A}\right) + \tan^{-1}\left(\frac{B_j}{A}\right)$$

and $I_a(t)$ is the total armature current.

$$K_1 = \frac{\beta_j}{L_a (A^2 + B_j^2)}$$

$$K_2 = \frac{\sqrt{(\beta_j - A)^2 + B_j^2}}{L_a B_j \sqrt{A^2 + B_j^2}} .$$

Differentiating Equation (8) and setting it equal to zero, the time at which peak current occurs is given by:

$$t_p = \frac{1}{B_j} \tan^{-1}\left(\frac{B_j}{A - \beta_j}\right) . \tag{9}$$

Combining Equations (8) and (9) we obtain the peak incremental current in terms of the generator parameters and the fault resistance:

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[10] *Scorgie, D. G., op. cit., Equation (30)*

$$i_a(t_p) = E_o' \left[K_1 + K_2 e^{-\frac{A}{B_j} \tan\left(\frac{B_j}{A-\beta_j}\right)} \sin\left(\tan^{-1}\frac{B_j}{A}\right) \right] \quad (10)$$

The reciprocal of the term in the square brackets of Equation (10) has the dimension of resistance. Since its form is not simple, it is best to determine the relative importance of the parameters by calculating families of curves using a practical set of parameters and varying one of them. It is hoped in this way to be able to understand better what factors must be considered. This is in contrast to certain recent attempts which have been made to find a correlation between transient resistance of a d-c generator and the product horsepower times speed.[11] The correlation was poor at best, having spreads of up to 50 percent in the data. But more important, such a correlation, even if good, would not lend much insight into the physical factors involved. The attempt in this report has been to express the current transient in terms of factors which can be easily measured, and which can be calculated from the physical structure of the machine.

DETERMINATION OF CONSTANTS OF GENERATOR

Resistance Values

Six constants of the generator must be determined before solving Equation (8). Of these, two are the resistances of the field and armature circuits which in the main are measured by well known methods. The resistance of the armature circuit is far from constant, but changes greatly as a function of load for small currents. The change is connected with commutator film and surface polish of the carbon brush.[12] At low-current densities the resistance of the carbon-copper contact is high and is a function of time. This portion of the resistance drops rapidly to a low value at high-current density. The value of R_a used in this report is always measured at full load.

Demagnetization Constant

D_a was defined earlier in this report as $\frac{\partial e_g}{\partial i_a}$. If the terminal voltage, E_t, is expressed as a function of the armature and the field currents,[13] the expression may be differentiated partially with respect to armature current giving $\frac{\partial E_t}{\partial i_a} = \frac{\partial e_g}{\partial i_a} - R_a$. Adding armature resistance to both sides gives the constant D_a. The above partial derivatives can be approximated by incremental measurements, and the transients calculated in this report used values obtained in this manner. For simple shunt-wound generators this leads to no difficulty, but for compensated machines trouble is encountered in the region of small armature currents. In this region the changes of brush contact resistance with current density are on the order of magnitude of D_a itself and thus introduce prohibitively large errors unless great care is taken.

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[11] *AIEE Subcommittee on D-C Machinery, "Maximum Short Circuit Current of D-C Motors and Generators," AIEE Technical Paper No 50-23, November 1949*

[12] *Soper, P. F., "Cause of Selective Action with Carbon Brushes," BEAMA Journal, 56, 263-266, August 1949*

[13] *Van Valkenburg, loc. cit.*

Slope of Saturation Curve

K_s is the slope of the generator saturation curve for a constant armature current. Again the value may be obtained either by differentiating an analytical expression for terminal voltage, this time with respect to field current, or by direct measurements if the machine is available. Good results are obtained only when D_a and K_s are evaluated in the neighborhood of the initial operating conditions.

Field Self-Inductance

L_f is defined as 10^{-8} times the partial derivative of field-flux linkages with respect to field current. The field flux is related to the effective air-gap flux by a leakage coefficient which in aircraft generators is quite large and must be taken into account. If the generator is available L_f may be obtained by direct measurement in either of two ways. An oscillographic record may be made of the response of the field current to a step change of resistance with constant excitation, or a sinusoidal resistance change[14] can be introduced and voltage and current phase relationships observed as a Lissajou figure. The latter method is to be preferred but a sinusoidally variable resistance is not generally available.

Armature Circuit Self-Inductance

Methods for calculating L_a from a knowledge of the physical characteristics of the generator have not proven outstandingly accurate, and no attempt has been made to improve these calculations. Indeed it is clearly recognized that L_a is variable and probably the treatment of it as a constant is the weakest assumption which has been made. The approach here will be merely to define L_a and indicate the manner in which it was measured. As previously stated L_a is the incremental self-inductance of the entire armature circuit, including commutating and compensating windings, at the initial operating condition. In a previous report[15] the terminal voltage transient was expressed as a function of time upon removal of load from a regulated generator. Two discontinuities occur in the voltage transient, one at t = 0 and a second when the load current reaches zero. Both are primarily discontinuities in the $L_a \frac{di_a}{dt}$ voltages of the armature circuit. To a very good approximation L_a is given by dividing the step function change of terminal voltage at the instant load removal is initiated, by the rate of change of load current. It is recognized that L_a is influenced by eddy current paths in the field poles and rotor, and is hence a function of the speed of flux variations. However the checks between empirical and calculated transients appear sufficiently close to substantiate the measurements of L_a made in this fashion.

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[14] *Scorgie, loc. cit.*

[15] *Scorgie, loc. cit.*

APPLICATION TO EXISTING GENERATORS

The equations derived above have been applied to two aircraft generators; generator No. 1 is an obsolete type with a simple shunt field rated at 75 amps, 30 volts, while generator No. 2, which has been widely used, has compensating and commutating fields and is rated at 200 amps, 30 volts.

Figures 2 through 7 show families of curves calculated for generator No. 1 with various values of fault resistance. In Figure 2 it was assumed that there was no regulator, that the generator was self-excited and that the initial load current was 75 amperes, corresponding to full load. In Figure 3 the same conditions, were assumed but with separate excitation. In Figure 4 it was assumed that a carbon-pile voltage regulator was connected, that the generator was self-excited and initial load was 75 amperes. Figures 5, 6, and 7 are the corresponding cases calculated assuming the initial load equalled zero. The constants for generator No. 1 used in these calculations were as follows:

Initial Load	L_a	R_a	K_s	D_a	L_f	R_f	R_s
0	2.5×10^{-4}	.08	15.5	-.080	.55	21.3	4.16
Full	2.5×10^{-4}	.08	8.0	-.113	.25	10.1	4.16

Each figure also shows the corresponding measured transient-load currents and the agreement is reasonably good in each case.

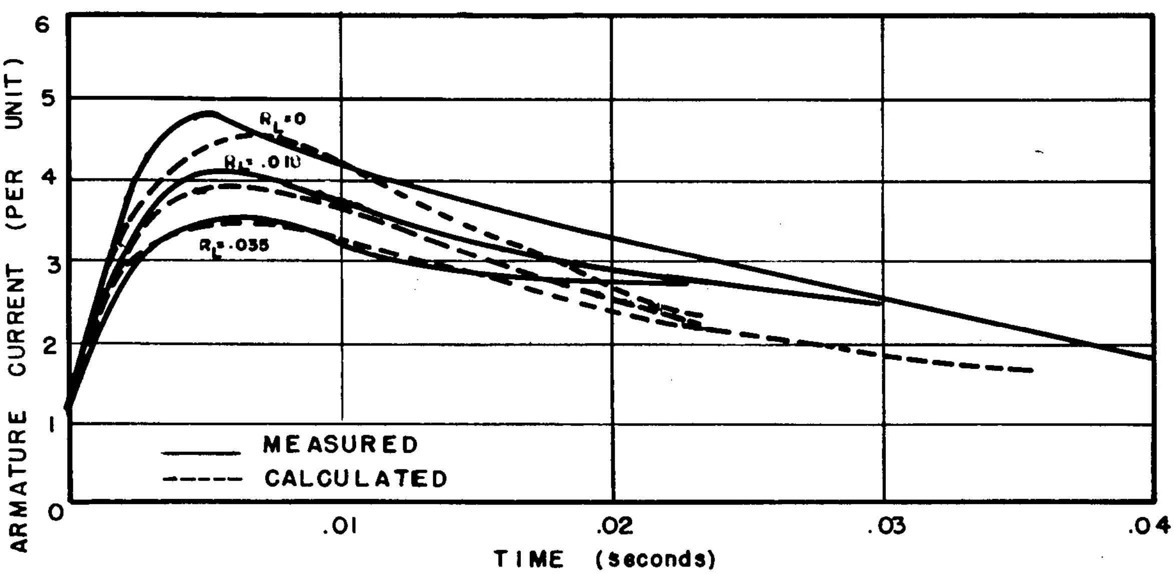

Figure 2 - Generator No. 1 (no regulator, self-excited, full load)

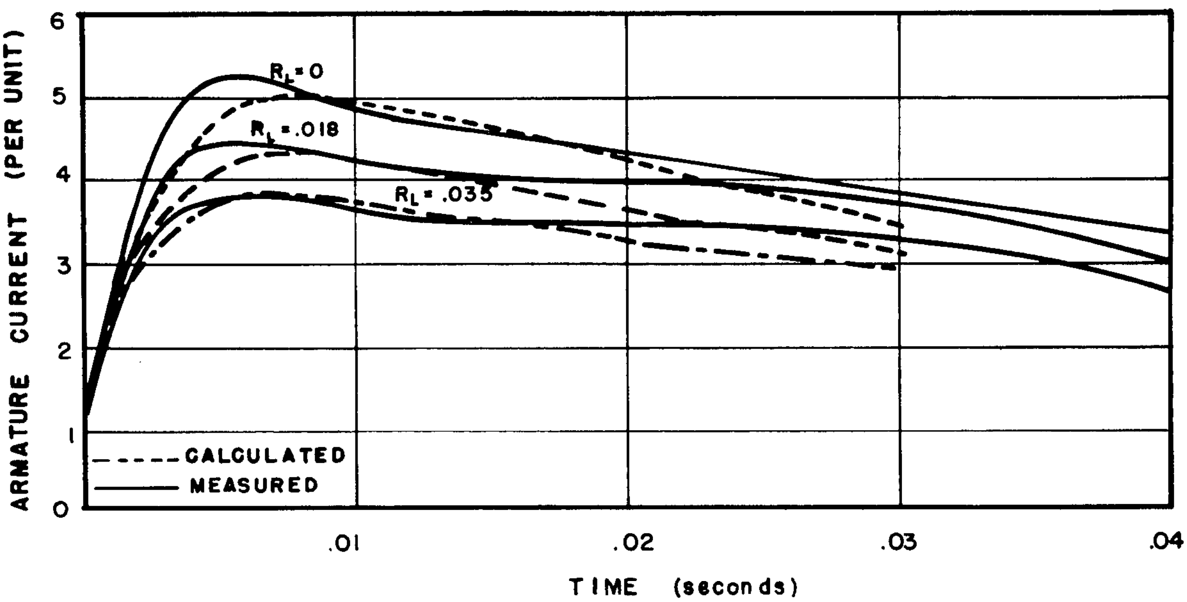

Figure 3 - Generator No. 1 (no regulator, separate excitation, full load)

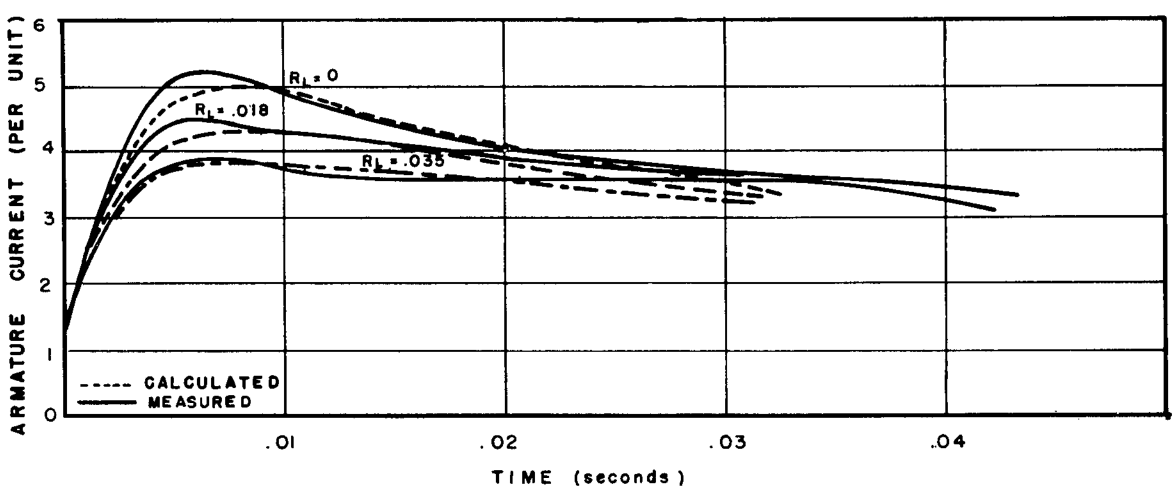

Figure 4 - Generator No. 1 (with regulator, self-excited, full load)

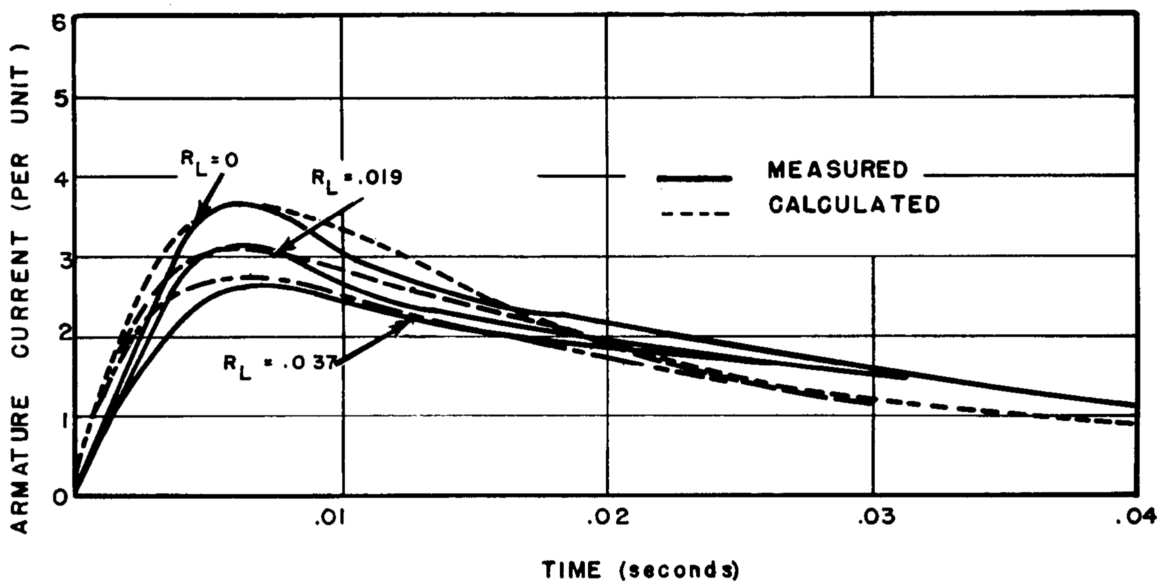

Figure 5 - Generator No. 1 (no regulator, self-excited. load zero)

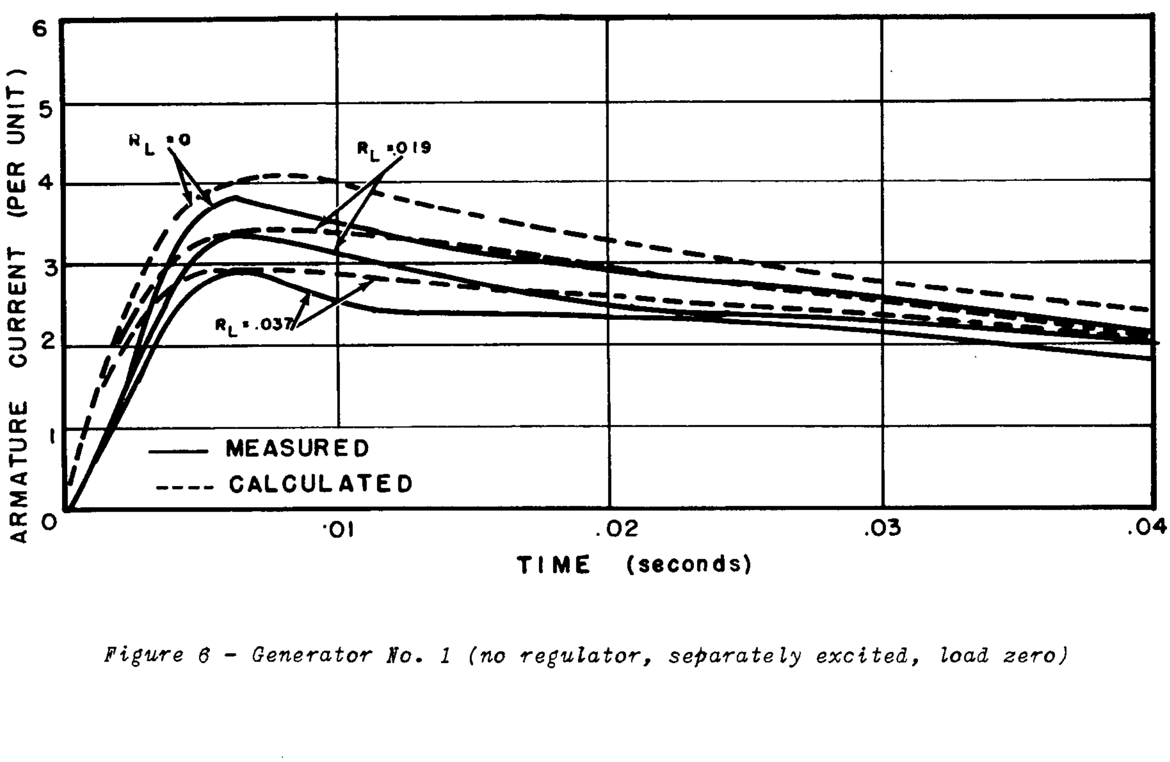

Figure 6 - Generator No. 1 (no regulator, separately excited, load zero)

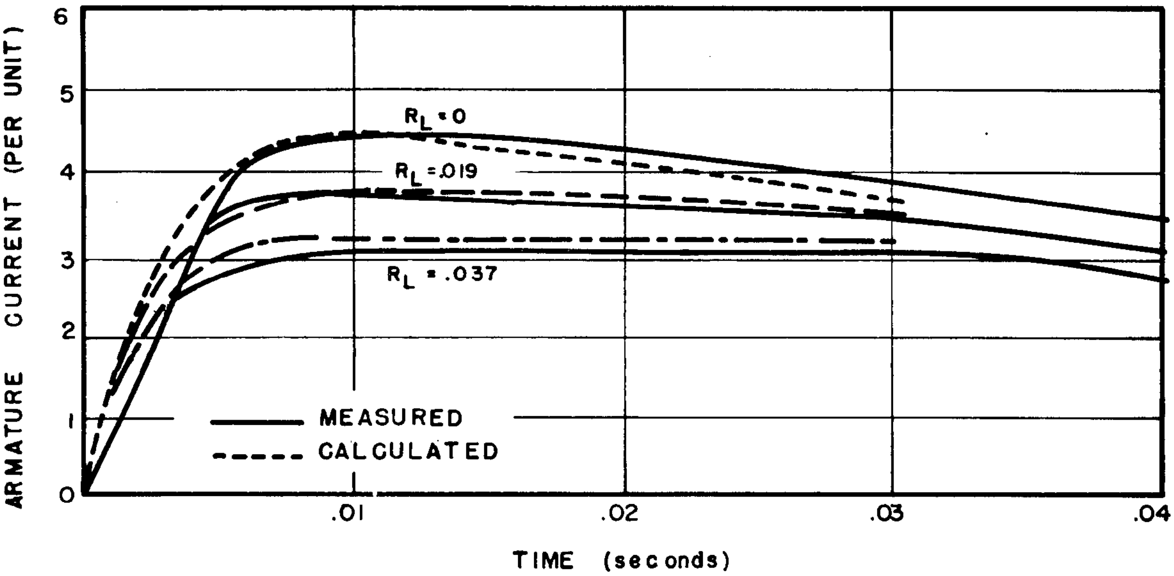

Figure 7 - Generator No. 1 (with regulator, self-excited, load zero)

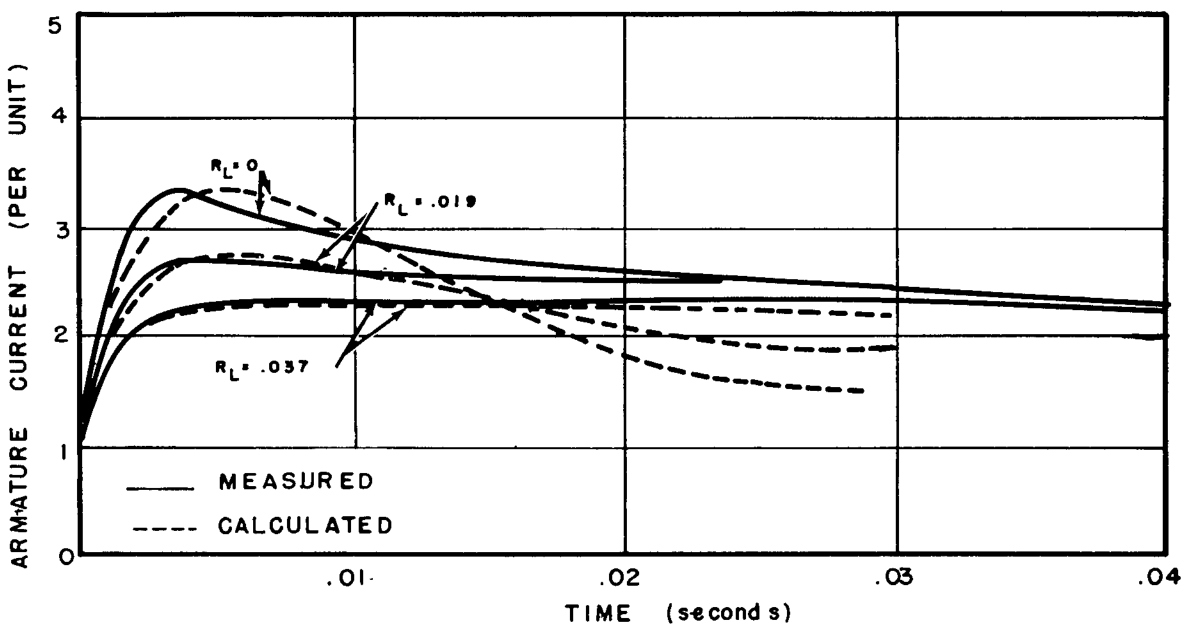

Figure 8 - Generator No. 2 (with regulator, self-excited, full load)

Figure 8 shows a typical comparison between measured and calculated transients with the compensated generator No. 2. The constants, such as L_f and D_a, for this machine are much smaller (by a factor of 5 to 10) than with generator No. 1, yet the absolute magnitude of the magnetic hysteresis in both generators is nearly the same. With the simple shunt machine hysteresis could account for a maximum error of ±8 percent in the value of D_a, whereas with the compensated generator hysteresis causes changes in D_a up to ±30 percent. It is reasonable then to obtain the better fault current predictions assuming linearity with generator No. 1. However at least two additional factors should be investigated in connection with the problem, namely the effects of interpole saturation and eddy currents upon effective armature inductance.

FACTORS AFFECTING TRANSIENT RESPONSE

The transient resistance, ΔR, of d-c generators has been defined in various ways by different investigators.[16,17] Linville defined it as the ratio of the initial generated voltage to the total peak surge current. In general, only zero resistance faults and zero initial load have been considered, so that the generated voltage is the total terminal voltage existing prior to the short circuit. From Equation 10 it can be seen that the ratio of peak i_a to E_o' is expressed in terms of the parameters of the generator and the fault resistance. Therefore this appears to be a fundamental relationship. E_o', it should be recalled, is that voltage which appeared across our shorting switch immediately before it was closed across all or part of the initial load resistance. Therefore transient resistance has been defined as the ratio of the instantaneous voltage change at $t = 0$ to the peak incremental armature current.

The fact that the peak change of load current rather than peak total current is fundamental explains in part why other investigators have found transient resistance to decrease with increased initial load current. The difficulty with specifying a unique transient resistance for a particular generator even by the use of Equation (10) is that the generator parameters change as a function of initial conditions. With a compensated machine such as generator No. 2 the parameters vary relatively little with initial conditions. If for instance the parameters are measured at 30 volts, both no-load and full-load, the constants will remain about the same except for a decrease of R_f. This results in an increased time constant for the shunt field which in some cases decreases the transient resistance slightly.

Transient resistance, ΔR, defined as in Equation 10 has repeatedly proven to contain a constant of the generator which is independent of fault resistance. If the fault resistance is subtracted from ΔR one gets the zero-fault transient resistance, $(\Delta R)_o$. Table 1 shows values calculated from a series of measured curves (Figures 1-6).

The parameters affecting transient resistance depend a great deal upon which condition is being analyzed, and in particular which term of Equation (10) dominates.

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16 *Linville, loc, cit.*

17 *Kaufmann, loc. cit.*

TABLE 1
Measured Transient Resistances

Regulator	Excitation	Initial Load	Fault Resistance, R_L	Measured ΔR	$(\Delta R)_0 + R_L$
No	Self	0	0	.111	.111
No	Self	0	.019	.128	.130
No	Self	0	.037	.151	.148
No	Separate	0	0	.105	.105
No	Separate	0	.019	.119	.124
No	Separate	0	.037	.143	.142
Yes	Self	0	0	.091	.091
Yes	Self	0	.019	.109	.110
Yes	Self	0	.037	.131	.128
No	Self	Full	0	.105	.105
No	Self	Full	.018	.122	.123
No	Self	Full	.035	.144	.140
No	Separate	Full	0	.095	.095
No	Separate	Full	.018	.112	.113
No	Separate	Full	.035	.129	.130
Yes	Self	Full	0	.095	.095
Yes	Self	Full	.018	.110	.113
Yes	Self	Full	.035	.127	.130

Expressing K_1 of Equation (10) in terms of the generator parameters it is seen to be independent of any inductance terms. However if the transcendental term dominates as for instance with a self-excited generator with no regulator and a small fault resistance, the transient resistance is critically dependent upon the mutual inductance between the shunt field and the armature circuit. If the constant term of Equation (10) dominates, such constants as the armature and fault resistances and the demagnetization coefficient are the determining factors. In the latter case if the generator is self-excited, the field resistance and the slope of the saturation curves are important. Experiments have been performed varying the field time constant when the generator was separately excited. The effect upon the fault current transients was virtually negligible.

Generator speed has been observed[18] to affect the transient resistance, always increasing it with increased speed. For cases when the transcendental term of Equation (10) dominates this is due primarily to the increase of the mutual inductance coefficient and the decrease of the shunt field circuit time constant with speed.

[18] *McClinton, loc. cit.*

A most pronounced effect upon transient peaks and therefore transient resistance is due to a change of the self-inductance of the armature circuit. Figure 9 shows a typical set of calculated curves in which reducing the self-inductance of the armature circuit by 50 percent increases the transient peak current by 18 percent. In the same figure the effect of decreasing the magnitude of K_s by 35 percent and D_a by 25 percent is illustrated. In general a decrease of the magnitude of either D_a or K_s raises the calculated steady-state fault current, and to a lesser extent raises the peak current surge.

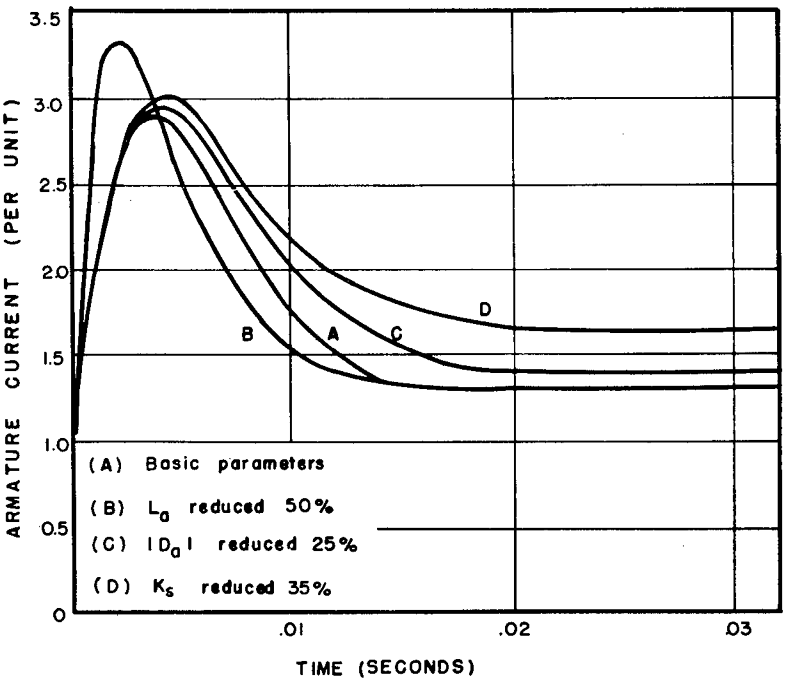

Figure 9 - Calculated variation of parameters, generator No. 2 (no regulator, self-excited, full load)

It was stated earlier that the generated voltage and field flux decrease markedly by the time the peak fault current is reached. One quantitative example will suffice to illustrate this point. Consider the zero-fault transient, plotted in Figure 2, in which the values of D_a = .080 and K_s = 15.5 were used. Solution of the equations show that at time of the peak fault current i_a = 270 amperes and i_f = 0.82 ampere. From Equation (1') it follows that the generated voltage has decreased 9.0 volts or 30 percent of its original value. The fact that by use of a constant flux leakage coefficient correct fault currents were calculated is presented as proof that the shunt field flux decreased proportionally to the generated voltage.

SUMMARY

Assuming linearity of incremental inductance coefficients, demagnetization due to load current, and saturation curve slope, a set of two simultaneous, linear, differential equations have been written which satisfactorily describe transient fault currents

up to and past the peak surge. The solution to these equations indicates that a reasonable definition for transient fault resistance of a generator is the ratio of the step function change of terminal voltage at the instant of fault to the maximum incremental load current. Nevertheless by this definition the effect of initial conditions, excitation and the presence of a voltage regulator must be considered. Generated voltage in typical cases has decreased about 16 percent at the peak current surge, but decreases as much as 30 percent have been noted. The voltage regulator is found to decrease transient resistance, and this decrease is reasonably well accounted for by the differential equations which have been solved.

The values of mutual inductance used everywhere in this report were derived assuming that the leakage flux coefficients remained constant throughout each transient. Results appear to justify this assumption in the case of the simple shunt generator. However the presence of interpoles and compensating windings introduce some inaccuracy in the results. Further investigation will be necessary to determine whether the difference is due to magnetic hysteresis, changing leakage-flux coefficient, or the changing coefficients of self and mutual inductance.

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APPENDIX

NOMENCLATURE

R_a	=	The d-c resistance of the armature circuit consisting of the armature winding and, if any, the compensating and commutating winding.
R_L	=	The total external fault resistance.
$(R_f)_o$	=	The shunt field-circuit resistance prior to the application of fault.
R_s	=	The minimum field-circuit resistance with carbon-pile resistor at its least value.
R_f	=	The field-circuit resistance after fault (R_s when a regulator is connected, and $(R_f)_o$ with none).
$(I_a)_o$	=	Load current prior to fault.
i_a	=	Incremental change of armature current.
i_f	=	Incremental change of shunt field current.
e_g	=	Incremental change of generated voltage from that existing prior to fault.
E	=	Constant excitation voltage with separately excited field.
E_o	=	Terminal voltage prior to fault.
E_o'	=	$E_o - (I_a)_o R_L$ = step driving voltage introduced at $t = 0$.
L_a	=	Incremental self-inductance of the armature circuit.
L_f	=	Incremental self-inductance of the shunt field circuit.
D_a	=	$\frac{\partial e_g}{\partial i_a}$ with constant field current.
K_s	=	$\frac{\partial e_g}{\partial i_f}$ with constant armature current.
ϕ_f	=	Flux linking shunt field winding.
M	=	$N_f \frac{\partial \phi_f}{\partial i_a} \times 10^{-8}$ with constant field current.
ΔR	=	The transient resistance of the generator.
$(\Delta R)_o$	=	Transient resistance for zero resistance fault.

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